UNIVERSALLY CONSERVED QUANTITIES

by

J. R. Christman

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   b. Fermions, Yes; Bosons, No

6. Work These Problems

Acknowledgments
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**Input Skills:**

1. State the family and family group to which a particle belongs (MISN-0-274).
2. Explain what is meant by saying a property is conserved (MISN-0-275).

**Output Skills (Knowledge):**

K1. For each strong-stable particle, give its charge, baryon number, electron number, muon number, and lepton number.
K2. State the conservation laws which ensure the stability of the: (a) proton; (b) electron; (c) electron neutrinos and antineutrinos; and (d) muon neutrinos and antineutrinos.

**Output Skills (Problem Solving):**

S1. Given a decay or interaction in the form \[ A \rightarrow B + C + D, \] show that it satisfies the universal conservation laws.
S2. Given a list of possible interactions, identify those that cannot occur because a universal conservation law would be violated.

**External Resources (Required):**


**Post-Options:**


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1. Introduction
This module deals with additive quantum numbers associated with conserved quantities, and the specific values assigned to various particles.

2. Charge
2a. \( Q \) is an Interaction Strength. Charge determines the strength of the electromagnetic field produced by the particle. It also determines the force on the particle if it is put into the electromagnetic field produced by other particles.

2b. Particle and Antiparticle \( Q \). The charge associated with each particle is listed in the table in “The Strong-Stable Particles,” MISN-0-274. The charge on an antiparticle is equal in magnitude to the charge on the corresponding particle, but opposite in sign.

2c. \( Q \) is Integer Multiple of \( e \). The magnitude of the charge on any particle so far observed is a small integer times the charge \( e \) on the proton. All particles listed have charge \( +e \) or \( -e \) but other fundamental particles have greater amounts of charge. The \( \Delta^{-} \), for example has charge \( -2e \).

2d. \( Q \) Conservation: Consequences. Charge is conserved in the sense that the sum of all charge entering an interaction is always the same as the sum of all charge leaving. The sum must take into account the sign of the charge. For example, in the interaction
\[
e^+ + e^- \rightarrow \gamma + \gamma,
\]
there is zero net charge both before and after the interaction. Note that the identity of the particles may change but the total charge does not.

2e. Conservation of \( Q \) and \( E \); Consequences. We shall see later that if a single particle undergoes decay, the sum of the masses of the particles in the final state of the system must be less than the mass of the original particle. This is a consequence of Conservation of Energy. The electron is stable because energy and charge are conserved and there are no charged particles with mass less than the mass of the electron. The positron (antielectron) is stable for the same reason.

3. Baryon Number
3a. \( B \) Assignments. Each baryon is assigned the baryon number \( B = +1 \), each antibaryon is assigned \( B = -1 \), and all other particles are assigned \( B = 0 \).

3b. \( B \) Conservation. The net baryon number is conserved in an interaction in exactly the same manner that net charge is conserved. For example, in the interaction
\[
n + p \rightarrow \Lambda^0 + n + K^+ + \pi^0,
\]
a net baryon number of +2 on the left (\( B = 1 \) for the neutron, \( B = 1 \) for the proton) and a net baryon number of +2 on the right (carried by the Lambda and neutron).

3c. \( B \) In Baryon Production. The net number of baryons in the world can increase only if there is a like increase in the number of antibaryons.

3d. Conservation and Proton Stability. Baryon number conservation (in conjunction with energy conservation) is responsible for the stability of the proton and the antiproton: the proton is the lightest baryon.

4. Lepton Numbers
4a. Electron No. and Stability of the Electron’s Neutrino. Electrons and electron neutrinos are assigned electron number \( +1 \), positrons and electron antineutrinos are assigned \( -1 \), and all other particles are assigned \( 0 \). Electron number is conserved in all interactions in precisely the same way as baryon number and charge. Simultaneous conservation of electron number and energy explains the stability of the electron neutrino and electron anti-neutrino.

4b. Muon No. and Stability of the Muon’s Neutrino. In a like manner, the muon (\( \mu^- \)) and its neutrino are each assigned muon number \( +1 \), the antimuon (\( \mu^+ \)) and the muon antineutrino are assigned \( -1 \), and all other particles are assigned \( 0 \). Muon number is conserved in the same manner as electron number. This conservation law, along with
energy conservation, accounts for the stability of the muon neutrino and antineutrino.

4c. Tauon No. and Stability of the Tau’s Neutrino. In a like manner, the tau (τ⁻) and its neutrino are each assigned tau number +1, the antitau (τ⁺) and the tau antineutrino are assigned −1, and all other particles are assigned 0. Tau number is conserved in the same manner as muon and electron number. This conservation law, along with energy conservation, accounts for the stability of the tau neutrino and antineutrino.

4d. Lepton Number. Another quantum number which is widely used is lepton number. Leptons (e⁻, μ⁻, τ⁻, νₑ, ν₈, ν₉) are assigned +1, antileptons (e⁺, μ⁺, τ⁺, νₑ, ν₈, ν₉) are assigned −1, and all other particles are assigned 0. It is obvious that the validity of individual conservation laws for electrons, muons, and taus implies a conservation law for leptons.

5. Family Number Conservation

5a. Mesons, Photons: No. There are no analogous conservation laws for mesons or for photons. These particles can be created or destroyed in any number, provided other conservation laws, such as the energy conservation law, are not violated.

5b. Fermions, Yes; Bosons, No. Family number conservation laws exist for fermions but not for bosons. These conservation laws are linked to spin by the laws of relativistic quantum mechanics (fermions have half-integral spin while bosons have integral spin).

For fermions, there exist negative energy states, separated from positive energy states by a gap of magnitude $2mc^2$. The vacuum state is described as one in which all negative energy states are filled and no positive energy states are filled. In this widely accepted model, the particles in the negative energy states are said to be unobservable. When enough energy is supplied to an (unobservable) electron in a negative energy state, it jumps up to one of the empty positive energy states and thereby becomes observable. The empty negative energy state it left behind (a “hole” in the negative energy states) acts like a positively charged particle and is an observable positron.

For example, in this model any positron is a hole in the all-pervasive “sea” of unobservable negative energy electrons. In the diagram below, energy is plotted vertically. There are no states between $-mc^2$ and $+mc^2$. All energies below $-mc^2$ and above $+mc^2$ are possible states that electrons can occupy.

<table>
<thead>
<tr>
<th>Energy</th>
<th>State Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E = +mc^2$</td>
<td>Vacuum = all states empty. Electron = 1 state filled</td>
</tr>
<tr>
<td>$E = 0$</td>
<td></td>
</tr>
<tr>
<td>$E = -mc^2$</td>
<td>Vacuum = all states filled. Positron = 1 state unfilled</td>
</tr>
</tbody>
</table>

No negative energy states exist for the bosons. However, boson antiparticles can be defined through the operation of charge conjugation, which is discussed in a later lesson. The force property of invariance under charge conjugation allows one to deduce properties of boson antiparticles from the corresponding particle properties.

6. Work These Problems


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