RELATIVISTIC MOMENTUM: PARTICLE DECAYS

by
Peter Signell

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Title: Relativistic Momentum: Particle Decays
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Input Skills:
1. Work simple conservation of momentum problems (MISN-0-14).
2. Use the relativistic form for total energy in conservation of energy problems (MISN-0-23).
3. Expand the square root function in a Taylor series about a given point (MISN-0-4).

Output Skills (Knowledge):
K1. Reduce the expression for relativistic momentum to its non-relativistic form, using the general expression for Taylor’s Series for the expansion of a function about a point.
K2. Show that \( \vec{F} = m\vec{a} \) is generally valid only for \( v^2 \ll c^2 \).

Output Skills (Problem Solving):
S1. Given a particle’s rest mass and velocity, calculate its relativistic momentum and energy.
S2. Use conservation of energy and momentum to work decay problems, 1 particle \( \rightarrow \) 2 particles, in the center of mass frame.

External Resources (Required):

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New authors, reviewers and field testers are welcome.

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1. Readings Alternatives: Choose One


1b. Brief Answers.

2. When \( v = \frac{1}{\sqrt{2}} \) c

\[
E_{\text{total}} = \sqrt{2}m_0c^2
\]
\[
E_{\text{kin}} = (\sqrt{2} - 1)m_0c^2
\]

15. Electron: 0.788 MeV, 0.761 c
Proton: 938.3 MeV, 1 \( \frac{c}{\sqrt{2}} \)

22. \( \vec{p}_1 = \vec{p} \)
\( \vec{p}_2 = -\vec{p} \)

where
\[
p = \frac{c}{2m_0} \left( m_0^4 + m_1^4 + m_2^4 - 2m_0^2m_1^2 - 2m_0^2m_2^2 \right)^{1/2}
\]

1c. Alternative 2: RR. Read in RR: Sections 3.2, 3.3, and 3.4 with the exception of those areas in 3.4 pertaining to electric and magnetic fields. Optional Problems: 20, 24, and 29 on pp. 106-109.

1d. Brief Answers.

20. \( 4.42 \times 10^{-36} \) kg; \( 2.208 \times 10^{-32} \) kg.

24. b. 0.511 MeV
   c. 938 MeV.

29. a. \( \frac{7}{12} M_0c \)
   b. \( \frac{1}{5} M_0c \)
   c. \( \frac{32}{12} M_0c^2 \)
   d. \( \frac{34.29}{12} M_0 \)
   e. \( \frac{0.71}{12} M_0c^2 \)

1e. Alternative 3: WSM. Read in WSM: Sections 3-1, 3-2, and 3-3, including the examples. For the purposes of this module it is not necessary to learn the derivations given in Section 3-1; only the results. Optional Chapter 3 problems: 1, 8, 25, pp. 83-4.

1f. Brief Answers.

1. 0.87 c
8. \( 8.3 \times 10^4 \)

25. \( 5.6 \times 10^{19} \) J for spaceship alone

2. Work These Problems

2a. Problem A. The rest mass of a \( \pi^+ \) ("pi-plus") meson is about 140 MeV/c\(^2\). If a \( \pi^+ \) is traveling at 0.8 c, compute its energy in MeV and momentum in MeV/c.

2b. mass of $\Lambda^0 = 1115.63 \text{ MeV}/c^2$

mass of $n = 939.5656 \text{ MeV}/c^2$

mass of $\pi^0 = 134.9739 \text{ MeV}/c^2$

Note: $(\text{momentum})^2 = k^2 v^2 m_0^2 = (k^2 - 1)m_0^2 c^2$ is a useful identity that you can easily prove.


You might also like to try Problem 12.28, AF.

Acknowledgments

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Answers to Problems

A. 233 MeV, 187 MeV/c.

B. Consv. of $E$: $m_\Lambda = k_n m_n + k_\pi m_\pi$

Consrv. of $\vec{p}$: $0 = k_n \vec{v}_n m_n + k_\pi \vec{v}_\pi m_\pi$

Squaring the second equation and using the identity in the Note,

(a) $(k_n^2 - 1)m_n^2 = (k_\pi^2 - 1)m_\pi^2$.

Squaring the first equation after solving it for $k_\pi m_\pi$:

(b) $k_\pi^2 m_\pi^2 = m_\Lambda^2 - 2m_\Lambda k_n m_n + k_n^2 m_n^2$.

Then putting (b) into (a) to eliminate $k_\pi$ gives:

$$k_n = \frac{m_\Lambda^2 - m_\pi^2 + m_n^2}{2m_\Lambda m_n}.$$
LOCAL GUIDE

The readings for this unit are on reserve for you in the Physics-Astronomy Library, Room 230 in the Physics-Astronomy Building. Ask for them as “The readings for CBI Unit 24.” Do not ask for them by book title.

PROBLEM SUPPLEMENT

Note: Problems 1, 3, and 4 also occur on this module’s Model Exam.

1. The rest mass of a $\Sigma^+$ (“sigma-plus”) particle is 1189.37 MeV/$c^2$. If it is traveling at speed 0.8000 $c$, compute its energy in MeV and its momentum in MeV/$c$.

2. Reduce the expression for relativistic momentum to its non-relativistic form, using the general expression for Taylor’s Series for the expansion of a function about a point.

3. In the decay of a $\Sigma^+$ particle at rest, to a proton and a $\pi^0$ particle, calculate the momentum of the proton in MeV/$c$.

\[
\begin{align*}
    m_p &= 938.2592 \text{ MeV/}c^2 \\
    m_{\pi^0} &= 134.9645 \text{ MeV/}c^2 \\
    m_{\Sigma^+} &= 1189.37 \text{ MeV/}c^2
\end{align*}
\]


4. Show that $\vec{F} = m\vec{a}$ is generally valid only for $v^2 \ll c^2$. 
Brief Answers:

1. 1982 MeV, 1586 MeV/c.

2. Taylor’s Series:
   \[ f(x) = f(0) + \frac{f'(0)}{1!} x + \frac{f''(0)}{2!} x^2 + \ldots \]
   
   \[ k(x) = (1 - x)^{-1/2} \text{ where } x \equiv v^2/c^2 \text{ for our case} \]
   
   \[ k'(x) = (1/2)(1 - x)^{-3/2} \]
   
   \[ k''(x) = (3/4)(1 - x)^{-5/2} \]
   
   \[ k(v^2) = 1 + (1/2)x + (3/8)x^2 + \ldots \]
   
   \[ k(v^2) = 1 + (1/2)(v^2/c^2) + (3/8)(v^4/c^4) + \ldots \]
   
   mom. = \( k v m_0 = m_0 v + (1/2)m_0(v^3/c^2) + \ldots \)
   
   Then if \( v^2 \ll c^2 \) we can neglect the second term and get:
   
   \[ \text{mom.} = m_0 v, \quad \text{for } v^2 \ll c^2. \]

3. As in Problem B, derive:
   
   \[ k_p = \frac{m_\Sigma^2 - m_\Sigma^2 + m_p^2}{2m_\Sigma m_0 p} \]
   
   and find the numerical value of the momentum.

4. Newton’s Second Law, verified relativistically, is:
   
   \[ \vec{F} = \frac{d\vec{p}}{dt} \]
   
   but then:
   
   \[ \vec{F} = \frac{d(m\vec{v})}{dt} \]
   
   \[ = m\vec{a} + \vec{v} \frac{dm}{dt} \]
   
   where \( m \equiv km_0 \)
   
   \( \neq m\vec{a}, \) unless \( v^2 \ll c^2 \) so \( dm/dt = 0. \)

MODEL EXAM

1. See Output Skills K1-K2 on this module’s ID Sheet. One or both of these skills, or none, may be on the actual exam.

2. See this module’s Problem Supplement, Problem 1.

3. See this module’s Problem Supplement, Problem 3.

4. See this module’s Problem Supplement, Problem 4.

Brief Answers:

1-4. See this module’s text and Problem Supplement.